



# A Graph Search-Based Trajectory Optimiser for Practical Wind-Optimal Trajectories

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**Abstract.** As air traffic management moves towards trajectory-based operations, route flexibility will increase and aircraft will be less constrained by the current fixed air traffic services route network and airspace structures. To gain maximum benefit from this increased flexibility, it will be necessary to plan so-called “wind-optimal” tracks that take account of wind, especially for medium- and long-haul flights. The Electronic Navigation Research Institute (ENRI) has developed a dynamic programming-based trajectory optimiser that can generate ideal wind-optimal trajectories for research purposes, but its high computational burden and the difficulty of applying operational constraints mean that it is not suitable for creating practical flight trajectories. We are therefore exploring alternative means of creating practical wind-optimal trajectories. In this paper, we describe a proof-of-concept program that represents possible aircraft lateral routes as a graph and calculates wind-optimal trajectories by a shortest-path search using fuel consumption or flight time as a metric instead of distance. We present preliminary results of track computation assuming constant altitude and airspeed in the north Pacific free route area, and show a sample qualitative comparison with Pacific Organised Track System tracks and an actual flight track that demonstrates the potential feasibility of our approach.

**Keywords:** Trajectory optimisation · Air traffic management · Trajectory-based operations · Shortest-path search · Graph search

## 1 Introduction

As air traffic management (ATM) moves towards trajectory-based operations (TBO), the flexibility of aircraft operators to plan flight routes that better match their objectives will increase. Already in Europe, blocks of free route airspace (FRA) are being created where operators may plan routes between FRA entry and exit points, either directly or via intermediate points, without any relationship to air traffic service (ATS) fixed route structures [1]. Most intra-European flights are of short duration and the emphasis of FRA appears to be on achieving the most “direct” route (that is, closest to Great Circle). However, as flight duration lengthens, the benefit of flying a “wind-optimal” trajectory, that is, a trajectory that takes account of forecast winds, rather than shortest route, increases. Not all operators have the capability to generate such flight plans at present,

but to exploit high routeing flexibility, we expect that the demand for wind-optimal trajectory generation capability will grow. A trajectory generator is required that can create trajectories that are not only close to optimal in terms of flight time or fuel cost, but can also take account of ATM constraints.

The Electronic Navigation Research Institute (ENRI) has developed a dynamic programming (DP)-based trajectory optimisation program that can compute ideal wind-optimal trajectories. However, the computation time can be several hours for long duration flights, and it is difficult to apply ATM or operational constraints, so it cannot be used for generating operationally feasible trajectories. We therefore decided to explore an alternative approach to creating wind-optimal trajectories that are usable in an operational environment, based on shortest-path graph search.

In the following, Sect. 2 describes our existing wind-optimal trajectory generator and its limitations for operational use. Section 3 describes our proof-of-concept trajectory generator based on shortest-path graph search, including its principles, implementation and initial validation results comparing generated tracks with Pacific Organised Track System (PACOTS) tracks and actual flight data. Section 4 presents conclusions and future works.

## 2 Ideal Trajectory Generation

Building on work carried out at Kyushu University [2], ENRI has developed a wind-optimal trajectory generation program based on dynamic programming that conducts an iterative search of a parameter space comprising lateral profile (downrange and cross-track distance), vertical profile (altitude) and air speed. The algorithm attempts to minimise a cost function that balances fuel consumption and flight time via a cost index parameter. The cost between points in the parameter space is computed using the EUROCONTROL Base of Aircraft Data (BADA) version 3 aircraft performance model and associated performance parameters [3], and uses atmospheric data (horizontal wind velocity, temperature, geopotential height) interpolated from gridded numerical weather forecast data. The authors have used the program in studies showing benefits from Dynamic Airborne Re-route Procedure (DARP) and trajectory-based operations, amongst others [4, 5].

Although this program is capable of generating ideal wind-optimal trajectory profiles, it is difficult to apply operational or air traffic management constraints; for example, constraining cruise altitude to a permissible flight level, executing step climb, avoiding a restricted airspace or forcing the aircraft to pass a certain waypoint. One method to apply such constraints is to modify the cost function to give a very high penalty if a constraint is exceeded, but this can lead to trajectory anomalies and is difficult to apply. A further issue is that the intermediate points on the ideal trajectory have higher positional precision than can be entered into aircraft on-board navigation systems and ground-based flight planning and air traffic management systems, or are admissible in environments such as oceanic airspace.

Another limitation is the computational requirement. The large parameter space means that the search is computationally intensive. (For a sample set of 147 flights from Tokyo Narita airport to Honolulu airport on 12 different days in 2017 (one day in

each month) with six different aircraft types, the average trajectory calculation time was 9h8m50 s using a cluster of 112 CPU hardware threads: 80 threads on four Intel Xeon E5-2660 2.20 GHz CPUs and 32 threads on one Intel Xeon CPU E5-2690 2.90 GHz CPU.) Furthermore, the speed of convergence to an optimal solution can depend on the aircraft type and winds. We have attempted to reduce the size of the parameter space by, for example, restricting the cross-range parameter space depending on the Great Circle distance (given that shorter flights are unlikely to deviate far from the Great Circle) but this converges the trajectory to a local optimum.

These limitations mean that although our trajectory optimiser is useful in research for analysing potential benefits from ideal trajectories, its use is impractical for use in an operational environment.

## 3 Trajectory Optimiser Based on Graph Search

### 3.1 Research Requirements

Part of our research concerns operations in the north Pacific: flights between East Asia and North America and parts of Oceania. The environment consists of a “free routing” area in the central north Pacific in which aircraft operate on User-Preferred Routes (UPR) or use Pacific Organised Track System (PACOTS) tracks which are computed daily by the US Federal Aviation Administration and the Japan Civil Aviation Bureau for certain city pairs and take into account forecast winds and lateral separation constraints. PACOTS and UPR tracks are based on a grid of points with a resolution of  $1^\circ$  latitude and  $5^\circ$  or  $10^\circ$  longitude. In addition, there is a set of fixed ATS routes between Japan and Alaska called the NOPAC (North Pacific) routes. Free route tracks can join or split from the NOPAC routes at certain points. For interface between oceanic and radar-controlled air spaces in Fukuoka and Oakland flight information regions (FIR), the oceanic portions of flight plan routes must start and end at designated “gateway” points. Furthermore, we are extending our research to examine free route areas in East Asian airspace, for example at high altitude in radar-controlled airspace [6].

We therefore require a trajectory generator that can generate optimal routes in oceanic and non-oceanic free route environments that can reflect operational constraints such as restricted air spaces, having to pass designated gateway points and limiting to certain cruise Mach numbers and flight levels to ensure separation in non-radar environments.

### 3.2 Approach and Implementation

To meet these requirements, we devised an initial approach based on the shortest-path search of a graph. Permissible aircraft lateral trajectories are represented as a graph comprised of nodes (vertices) and edges. We consider that this representation will make it relatively easy to express certain types of ATM restriction.

- A “free route” area such as the north Pacific free route area can be represented as a mesh of nodes at appropriate intervals bounded by a polygon. The mesh can be joined to other airspaces or ATS route structures by boundary points (i.e. points along a free route airspace boundary such as gateway points).
- ATS routes are represented in the graph as edges between nodes that are defined waypoints along the route. Altitude and direction constraints can be expressed as properties associated with edges.
- Restricted airspaces such as military training areas and prohibited areas, or areas such as volcanic ash advisories (VAA) [7], can be represented by polygons that cause intersecting nodes and edges to be marked as unavailable (that is, effectively removed from the graph).

Constraints could be dynamic; that is, time-varying or even moving in the case of airspace restrictions.

Finding an “optimal route” then involves computing the shortest path through the graph between two points using a metric that represents a cost function rather than spatial distance. In our initial implementation, we use the well-known Dijkstra’s algorithm for the search. We are currently implementing a proof-of-concept program using the Python 3 programming language with the following characteristics.

- The graph is defined as a mesh in north Pacific airspace based on a grid of nodes at 1° latitude and 5° longitude intervals. Each node is connected to nodes in the two adjacent latitude columns. (Direct north/south paths are not defined.) The mesh may be connected to gateway points. Restricted airspaces are represented by polygons in which the mesh is not generated.
- Aircraft fly at constant true airspeed (TAS) and constant altitude.
- Atmospheric conditions (horizontal component of wind, temperature, pressure altitude) are interpolated from Global Spectral Model (GSM) numerical forecast data published by the Japan Meteorological Agency (JMA) [8], which has data at intervals of 0.5°-degree latitude and longitude and at pressure levels up to 100 hPa, and at 1° intervals above that. At present, data for a single forecast time are used for the computation (no time interpolation between forecasts).
- Aircraft performance is calculated using the EUROCONTROL BADA 3 model and performance database.

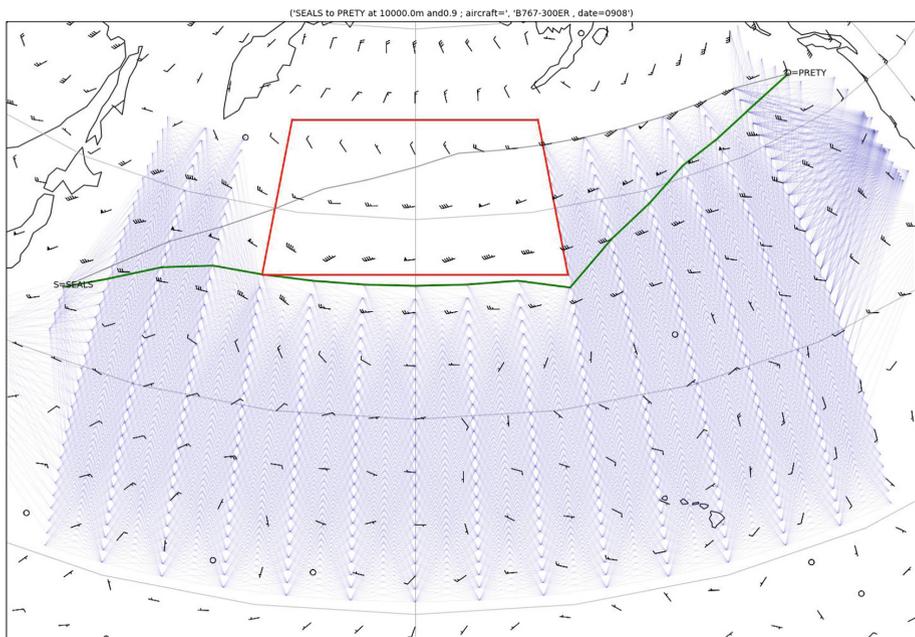
Initial specified conditions include start and end node, aircraft type, cruise TAS, cruise altitude, atmospheric forecast data set and initial aircraft mass. The optimal trajectory is then computed using Dijkstra’s algorithm with the “distance” along an edge between nodes being fuel consumption or flight time. Aircraft state parameters are calculated and integrated at 10 points along the edges.

### 3.3 Preliminary Results

Figure 1 shows some of the elements of the proof-of-concept trajectory optimiser. A mesh that roughly represents the north Pacific free route area is defined between 20° N 150°E and 50°N 130°W. This is linked to a set of gateway points east off the coasts of Japan and North America, including the points SEALS and PRETY. The red

polygon shows an area of airspace which is closed to traffic, hence the grid is not defined in this area. The forecast data set is for 8 September 2017 12:00 UTC ('nowcast' forecast). Horizontal wind direction and strength at 10,000 m altitude are shown in the figure by barbs. The thin grey line shows the shortest distance between SEALS and PRETY using the complete mesh (through the restricted airspace). The green line shows the computed minimum fuel route for a Boeing 767-300 aircraft cruising at 10,000 m.

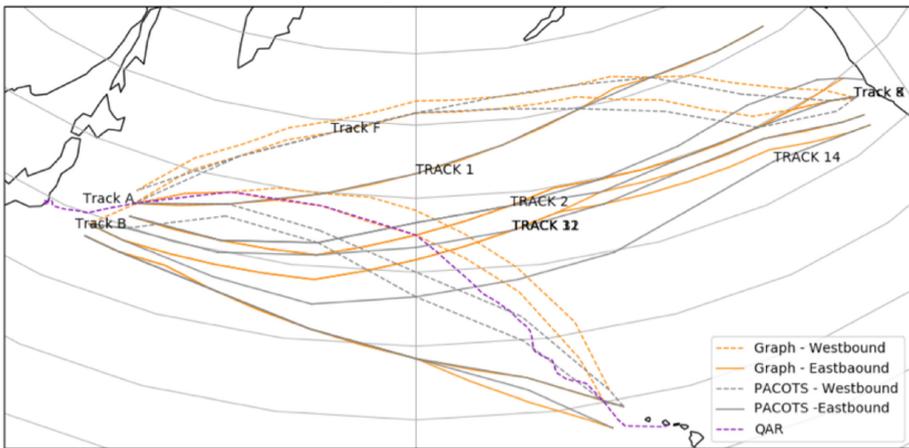
Regarding computation time, at present the bulk of the program run time is occupied by between mesh generation and search. On an Apple Mac Mini (Intel Core i5 2.8 GHz CPU with four hardware threads, 16 GB memory, Mac OS X 10.13.4) running a standard Python 3.6 interpreter and using the numpy numerical computing library [9], mesh generation takes on the order of two minutes, and minimum fuel track (MFT) and minimum time track (MTT) searches each take under six seconds.



**Fig. 1.** Sample computed result for eastbound flight between SEALS and PRETY

For validation, we are comparing the tracks generated by our proof-of-concept optimiser against PACOTS tracks computed for certain city pairs and against actual flight data obtained from Quick Access Recorder (QAR) on-board data recorders. We used the forecast data for the same time as that used to generate the corresponding PACOTS tracks. Figure 2 shows a sample result. Orange lines are those computed by our proof-of-concept optimizer between the same endpoints and for the same conditions (cruise flight level, aircraft type and weather forecast) as the PACOTS tracks, and

the grey lines show the corresponding PACOTS tracks. Overall, there can be said to be good qualitative agreement between the PACOTS tracks and those computed by our program. Note that PACOTS tracks are manually adjusted to ensure lateral separation between tracks. The purple line indicates the flight track of a Boeing 767-300 from Honolulu to Tokyo obtained from QAR data. The two optimal tracks computed by our program from Honolulu to Tokyo are north of the published PACOTS tracks. The actual flight starts following the more southerly of the Honolulu–Tokyo PACOTS tracks, but then deviates north from it and follows our computed track from approximately 180°E. (This flight was a DARP operation that changed its flight plan *en route* based on latest weather information.)



**Fig. 2.** Comparison between computed optimal tracks (“Graph”), PACOTS tracks (“PACOTS”) and actual flight data for a Boeing 767 ‘DARP’ from Honolulu to Tokyo (“QAR”)

### 3.4 From Proof-of-Concept to First Version

Following validation of the proof-of-concept, we will further develop the trajectory optimiser so that it can be applied to our research. As a first stage of development, we will extend the graph to be able to compute wind-optimal routes for certain city pairs of interest. This will include connecting the free route area mesh to airport nodes via gateways and ATS routes. NOPAC routes will also be added. We will also allow specification of Mach number as well as TAS, and interpolation between atmospheric numerical forecast data sets.

Instead of mesh generation being carried out in the program itself at runtime, we will separate the graph generation into a separate program and have the graph loaded as a file.

One issue is the integration interval. At present, we integrate fuel consumption over 10 intervals along an edge. Since the longitude mesh interval is 5°, the integration interval along an edge is at least 0.5° degrees longitude (varies between approximately

20NM an 30NM depending on latitude). In a more complex graph, however, the distance between nodes will vary. We will have to consider how to vary the integration distance and whether a variable distance will have any effect on the precision of the computation.

## 4 Conclusion and Future Research Topics

In this paper, we have described our approach and initial efforts to create a trajectory optimisation program that can generate wind-optimal trajectories that are operationally practical and can reflect ATM constraints. We define possible lateral flight routes as a graph and use a shortest-path graph search algorithm with fuel consumption or flight time instead of distance as the metric. A simple proof-of-concept prototype that optimises only the lateral trajectory with fixed altitude and cruise speed has demonstrated the generation of reasonable trajectories over a mesh that mimics free route airspace in the north Pacific.

As future research topics, potential longer term developments include the following:

- Adding step-climb, considering the constraint of achieving a minimum climb performance (typically minimum 1,000 ft/min rate of climb) during execution of the manoeuvre.
- Trade off flight time with fuel consumption (i.e. add Cost Index).
- Optimisation of cruise altitude and speed. This will potentially greatly increase the size of the search space, and we will have to consider how to deal with this; for example, using a hybrid approach employing genetic algorithms or other meta-heuristics instead of exhaustive search.
- Adding climb and descent phases.
- Specifying time constraints at nodes in order to apply to trajectory-based operations.
- Alternative shortest-path algorithms for better performance to avoid restricted airspaces etc. (e.g. the heuristically-guided A\* search).
- Improving the precision of computation for certain aircraft types by including the BADA 4 model and parameters.

ENRI would appreciate the cooperation of other researchers in exploring these issues.

**Acknowledgements.** The authors thank All Nippon Airways for providing Quick Access Recorder information and for their co-operating on research into DARP benefits. The authors thank Mr. Seiichi Nakahara for his great assistance in developing the proof-of-concept prototype program. The meteorological data are hosted on Kyoto University's RISH server, and the authors express gratitude to Kyoto University for providing this service to the research community.

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